

Full Length Research Paper

A Survey on the dispersion relation of ELF waves in the Ionospheric E –region Plasma

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In this paper, dispersion relation of ionic waves of O_2^+ and NO^+ is considered in the plasma of E – region ionosphere. First, we obtain dispersion relation of ionic waves in the presence of earth's magnetic field. With calculating refractive index of E – region plasma in any height, we obtained average frequency k in these heights. Finally, diagram of ionic frequency O_2^+ and NO^+ is drawn.

Keywords: Dispersion relation, Extra Low Frequency (ELF) wave, E-layer ionospheric plasma
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INTRODUCTION

Like other plasma media, existing waves in plasma of E layer ionosphere include ionic and electron waves (Milikh et al., 2008). Since electrons because of their light weight have more mobility, they have high frequencies from range of MHz type (Chen, 1974), and because we are eager to ELF waves then, we didn't consider electronic waves in studying dispersion relation of Extra Low Frequency (ELF) waves in E- region. It is predicted that range of ELF waves are attainable in ionic waves such as 10-100Hz or 62.86280 rad/s. we consider dispersion relation of ionic waves of existing ions O_2^+ and NO^+ in E-region (Chen, 1974) and to answer this question that ionic waves in E- region are fluctuated with ELF range or not we studied ionic- waves in plasma also ionic – sound waves, Alfvén waves and magneto – sonic waves are selected (Parrot et al., 2007).

The dispersion relation of ionic wave in E –region plasma

In this section, dispersion relation of ionic waves is calculated in the presence of earth's magnetic field. Equation of earth's magnetic field is as following:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{M_E}{r^3} (-2\cos\theta\hat{e}_\theta) \quad (1)$$

Here, $\theta = \frac{\pi}{2}$, λ , λ is longitude, r , radius of the earth

and $\hat{e}_r$, $\hat{e}_\theta$ are unit vectors in θ , r directions respectively (Gelant et al., 1373).

For simplicity, magnetic field is selected in form of $\vec{B} = B_r\hat{r} + B_\theta\hat{\theta}$ and then we resolve it without deviation from the whole issue, E and k are selected in the direction of the φ unit of the spherical coordinate. For the equation of motion, we have:

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Table 1. Refractive coefficient, wave number and density of O_2^+ and NO^+ ions in different height

n (refractive coefficient)	n(NO+) density in m ⁻³	n(O ₂ ⁺) density in m ⁻³	k(m ⁻¹)	h(Km)
230	2×10^{10}	5×10^9	2.2×10^{-3}	90
270	3×10^{10}	9×10^9	2.8×10^{-3}	100
460	6×10^{10}	5×10^{10}	4.8×10^{-3}	110
505	7×10^{10}	6×10^{10}	5.28×10^{-3}	120
508	8×10^{10}	7×10^{10}	5.31×10^{-3}	130

$$mm \frac{\partial \bar{V}}{\partial t} = -en(\bar{E}_\parallel + \bar{V} \times \bar{B}) - \bar{\nabla} P_e \quad (2)$$

With above thesis and

$$\bar{V} = V_r \hat{r} + V_\theta \hat{\theta} + V_\phi \hat{\phi}$$

Where:

$$-iMn_o \omega V_r = en_o(-B_\theta V_\phi) = -en_o B_\theta V_\phi \quad (3)$$

$$-iMn_o \omega V_\theta = en_o(-B_r V_\phi)$$

$$-iMn_o \omega V_\phi = en_o(E + V_r B_\theta - V_\theta B_r) - i\gamma_1 KT_i k n_1$$

From above equations, we have

$$V_r = \frac{\epsilon B_\theta}{iM\omega} V_\phi \quad (4)$$

$$V_\theta = \frac{\epsilon B_r}{-iM\omega} V_\phi$$

For the turbulence in electron density and as a result, this in the ion density consists of (Lehtinen1 and Inan1, 2008):

$$\begin{aligned} n_{1e} &= n_o \frac{\epsilon \phi_1}{KT_e} \\ i\omega n_1 &= n_o i k V_\phi \\ n_1 &= \frac{n_o i k}{i\omega} V_\phi \end{aligned} \quad (5)$$

With putting n_1 , V_θ , V_r in motion equation, we have:

$$\begin{aligned} -iMn_o \omega V_\phi &= n_o \epsilon i k \frac{KT_e n_1 k}{n_o \epsilon i \omega} V_\phi \\ &+ \frac{n_o \epsilon^2}{iM\omega} (B_\theta^2 + B_r^2) V_\phi - i\gamma_1 KT_i \frac{k n_o i k}{i\omega} V_\phi \\ -iM\omega^2 &= -ik^2 KT_e + \frac{\epsilon^2 B^2}{iM} - i\gamma_1 KT_i k^2 \\ \omega^2 &= \frac{-ik^2 KT_e}{-iM} + \frac{\epsilon^2 B^2}{M^2} - \frac{i\gamma_1 KT_i k^2}{-iM} \\ \omega^2 &= \frac{k^2 KT_e}{M} + \frac{\epsilon^2 B^2}{M^2} - \frac{\gamma_1 KT_i k^2}{M} \\ \omega^2 &= \Omega_e^2 + k^2 \left(\frac{KT_e + \gamma_1 KT_i}{M} \right) \end{aligned} \quad (6)$$

As one can see, the last equation fits with dispersion relation of electrostatic ionic waves.

Mathematical Solution

For solving problem, we perform as following:

1- We extract ionic and electronic temperatures from related diagrams (in heights of 90, 100, 110, 120

and 130 km).

2- We consider the same temperature for two ions O_2^+ and NO^+ (Bam and Trueman, 1380).

3- For calculating k, wave number, we perform in desired heights mentioned above as following:

$$\begin{aligned} \omega &= 2\pi\nu = 2\pi \frac{c}{\lambda} \\ \omega &= \frac{kc}{n} \end{aligned} \quad (7)$$

So for finding k, we need n, ω (refractive index) from above equation (Kelley, 1989). We put ω in an average frequency in the range of ELF waves about $2\pi \times 500$.

Then refractive index is calculated as following:

$$k = \frac{\mu_0 \epsilon_0 c^2}{B^2} \quad (8)$$

$$n = \sqrt{K}$$

In k equation with the first approximation we have ignored from number 1.

We calculate refractive index in relevant heights of E layer mentioned earlier v_s the density of ions and existing electrons in desired heights in E-region, we reached to the table (1) above:

As noted earlier, for easiness with selecting middle frequency 500 in equation 7, we calculate refractive index and k in E region plasma altitudes.

Now we extract ω with putting ionic and electronic temperature and k in either dispersion relation of ionic waves.

Sound – Ionic Waves

Sound – Ionic Waves equation is following form (Primdahl, Provide year; Chen, 1974).

$$\omega = k \left(\frac{KT_e + 3KT_i}{M} \right)^{\frac{1}{2}} \quad (9)$$

As we can see from above diagram (Figure 1), frequencies are in range of ELF waves. Also it is seen that with increase of altitudes, frequency will increase.

With respect to diagram, NO^+ ionic frequencies are more than the frequencies of O_2^+ ion species due to it

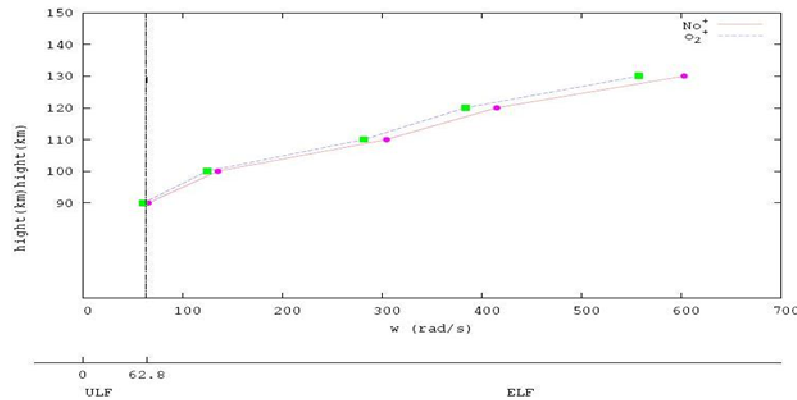


Figure 1. Sound – ionic frequencies of O_2^+ and NO^+ in different longitudes

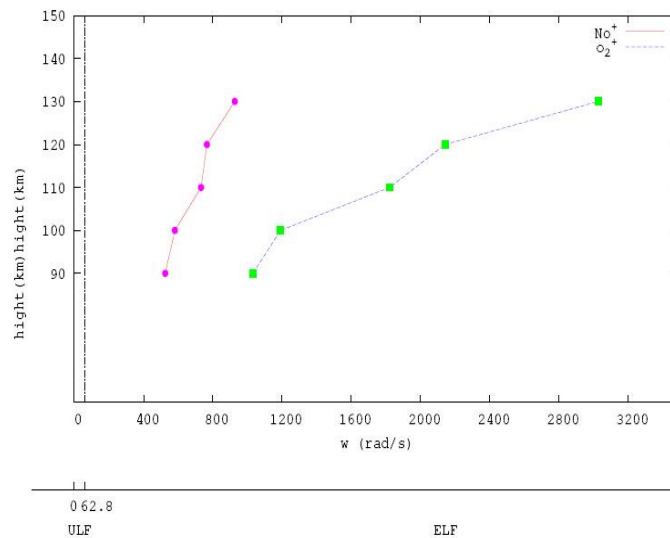


Figure 2. Alfvén wave frequencies of O_2^+ and NO^+ in different altitudes

more mass.

Alfvén Waves

With dispersion relation (Kelly, 1989; Chen, 1974).

$$\omega = k \frac{B_o}{(\mu_o \rho)^{\frac{1}{2}}} \quad (10)$$

Alfvén wave is the second type of ionic waves. We put k and density of different altitudes in equation to obtain frequency ω .

As can be seen from diagram (Figure 2), range of Alfvén waves is placed in range of ELF waves. As we can

observe with increase of altitude, frequency will increase. Increasing trend is like previous diagram. In addition to mass in denominator, density in denominator of dispersion relation is influential.

Magneto-Sonic Waves

The third type of ionic waves is magneto-Sonic waves. For calculating frequency, we replace Alfvén velocity and ionic velocity in different altitudes (Moore et al., 2006; Chen, 1974).

$$\omega = kc \left(\frac{V_s^2 + V_A^2}{c^2 + V_A^2} \right)^{\frac{1}{2}} \quad (11)$$

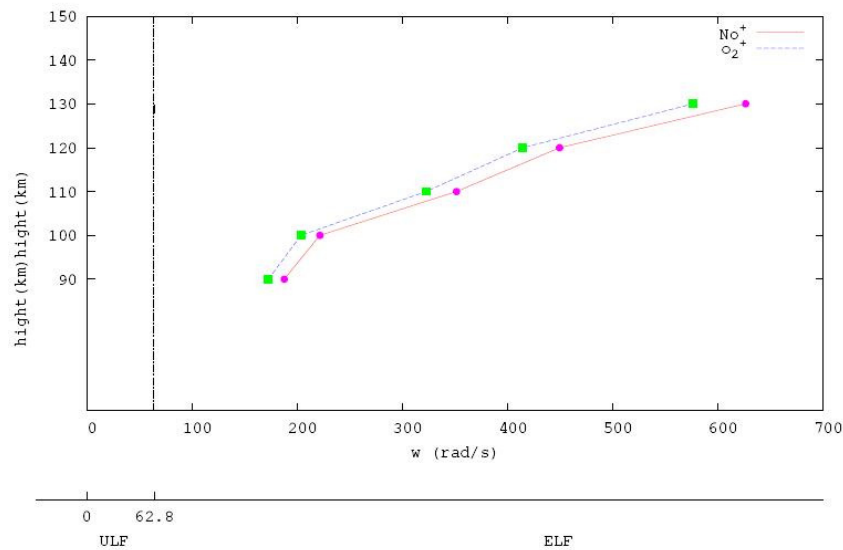


Figure 3. Magneto-Sonic frequencies of O_2^+ and NO^+ in different altitudes

As can be seen from diagram (Figure 3), magneto-sonic frequencies also are placed in range of ELF waves. The trends of variations in two previous diagrams are so that with increasing altitude, frequency of ions will increase.

CONCLUSION REMARKS

The results indicated that ionic, Alfvén and Magneto-Sonic waves are placed in range of ELF waves (10-1000 Hz). In result, we can study dispersion relation of ionic waves in E layer plasma in the range of ELF frequency. In drawn diagrams, NO^+ frequency is more than O_2^+ frequencies due to its lower mass value.

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