A Survey on the dispersion relation of ELF waves in the Ionospheric E–region Plasma

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In this paper, dispersion relation of ionic waves of $O_2^+$ and NO$^+$ is considered in the plasma of E–region Ionosphere. First, we obtain dispersion relation of ionic waves in the presence of earth’s magnetic field. With calculating refractive index of E–region plasma in any height, we obtained average frequency $k$ in these heights. Finally, diagram of ionic frequency $O_2^+$ and NO$^+$ is drawn.

Keywords: Dispersion relation, Extra Low Frequency (ELF) wave, E-layer 1ahra1heric plasma
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INTRODUCTION

Like other plasma media, existing waves in plasma of E layer ionosphere include ionic and electron waves (Milikh et al., 2008). Since electrons because of their light weight have more mobility, they have high frequencies from range of MHz type (Chen, 1974), and because we are eager to ELF waves then, we didn’t consider electronic waves in studying dispersion relation of Extra Low Frequency (ELF) waves in E-region. It is predicted that range of ELF waves are attainable in ionic waves such as 10-100Hz or 62.86280 rad/s. We consider dispersion relation of ionic waves of existing ions $O_2^+$ and NO$^+$ in E-region (Chen, 1974) and to answer this question that ionic waves in E–region are fluctuated with ELF range or not we studied ionic- waves in plasma also ionic – sound waves, Alfvén waves and magneto – sonic waves are selected (Parrot et al., 2007).

The dispersion relation of ionic wave in E–region plasma

In this section, dispersion relation of ionic waves is calculated in the presence of earth’s magnetic field.

Equation of earth’s magnetic field is as following:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{M_E}{r^3} (-2\cos\theta \hat{e}_\theta)$$ (1)

Here, $\theta = \frac{\pi}{2}$, $\lambda$ is latitude, $r$, radius of the earth

and $\hat{e}_r$, $\hat{e}_\theta$ are unit vectors in $\theta$, $r$ directions respectively (Gelant et al., 1373).

For simplicity, magnetic field is selected in form of $\vec{B} = B_r \hat{r} + B_\theta \hat{\theta}$ and then we resolve it without deviation from the whole issue, $E$ and $k$ are selected in the direction of the $\varphi$ unit of the spherical coordinate. For the equation of motion, we have:
Table 1. Refractive coefficient, wave number and density of \( \text{O}_2^+ \) and \( \text{NO}^- \) ions in different height

<table>
<thead>
<tr>
<th>n (refractive coefficient)</th>
<th>n(NO+) density in m(^{-3})</th>
<th>n(O(_2^+)) density in m(^{-3})</th>
<th>k(m(^{-1}))</th>
<th>h(Km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>230</td>
<td>(2 \times 10^{10})</td>
<td>(5 \times 10^9)</td>
<td>(2.2 \times 10^{-3})</td>
<td>90</td>
</tr>
<tr>
<td>270</td>
<td>(3 \times 10^{10})</td>
<td>(9 \times 10^9)</td>
<td>(2.8 \times 10^{-3})</td>
<td>100</td>
</tr>
<tr>
<td>460</td>
<td>(6 \times 10^{10})</td>
<td>(5 \times 10^{10})</td>
<td>(4.8 \times 10^{-3})</td>
<td>110</td>
</tr>
<tr>
<td>505</td>
<td>(7 \times 10^{10})</td>
<td>(6 \times 10^{10})</td>
<td>(5.28 \times 10^{-3})</td>
<td>120</td>
</tr>
<tr>
<td>508</td>
<td>(8 \times 10^{10})</td>
<td>(7 \times 10^{10})</td>
<td>(5.31 \times 10^{-3})</td>
<td>130</td>
</tr>
</tbody>
</table>

\[
\frac{\partial \vec{V}}{\partial t} = -en_0 (\vec{E}_f + \vec{V} \times \vec{B}) - \vec{V} \cdot \vec{P}_e
\]

With above thesis and

\[
\vec{V} = V_r \hat{r} + V_\theta \hat{\theta} + V_\phi \hat{\phi}
\]


Where:

\[
-\text{i} M n_0 \omega V_\phi = -en_0 (-B_\theta V_\phi) = -en_0 B_\phi V_\phi
\]

From above equations, we have

\[
V_r = \frac{\partial \phi}{\partial x} \quad V_\theta = \frac{\partial \phi}{\partial \theta} \quad V_\phi = \frac{\partial \phi}{\partial \phi}
\]

For the turbulence in electron density and as a result, this in the ion density consists of (Lehtinen1 and Inan1, 2008):

\[
n_i = \frac{n_0 \text{ion} \cdot \text{ion} \cdot \text{ion}}{\text{ion} \cdot \text{ion} \cdot \text{ion}}
\]

With putting n1, V\(_r\), V\(_\theta\), V\(_\phi\), in motion equation, we have:

\[
-\text{i} M n_0 \omega V_\phi = -n_0 \text{i} \frac{k T_i}{k T_e} \text{i} \frac{z^2}{\text{i} \frac{k T_e}{k T_i} \text{i} \frac{z^2}{\text{i} \frac{k T_i}{k T_e} \text{i} \frac{z^2}{\text{i} \frac{k T_i}{k T_e} \text{i} \frac{z^2}{\text{i} \frac{k T_i}{k T_e} \text{i} \frac{z^2}}}
\]

As one can see, the last equation fits with dispersion relation of electrostatic ionic waves.

**Mathematical Solution**

For solving problem, we perform as following:

1- We extract ionic and electronic temperatures from related diagrams (in heights of 90, 100, 110, 120 and 130 km).

2- We consider the same temperature for two ions \( \text{O}_2^+ \) and \( \text{NO}^- \) (Barn and Trueman, 1380).

3- For calculating k, wave number, we perform in desired heights mentioned above as following:

\[
\omega = 2\pi \nu = 2\pi \frac{L}{d}
\]

So for finding k, we need \( \rho \) (refractive index) from above equation (Kelley, 1989). We put \( \rho \) in an average frequency in the range of ELF waves about \( 2\pi \times 500 \). Then refractive index is calculated as following:

\[
k = \frac{\rho}{\lambda}
\]

\[
n = \sqrt{K}
\]

In k equation with the first approximation we have ignored from number 1.

We calculate refractive index in relevant heights of E layer mentioned earlier \( \nu_s \) the density of ions and existing electrons in desired heights in E-region, we reached to the table (1) above:

As noted earlier, for easiness with selecting middle frequency 500 in equation 7, we calculate refractive index and k in E region plasma altitudes.

Now we extract \( \omega \) with putting ionic and electronic temperature and k in both dispersion relation of ionic waves.

**Sound – Ionic Waves**

Sound – Ionic Waves equation is following form (Primdahl, 1997).

\[
\omega = k \left( \frac{KT_e + 3KT_i}{M} \right)^{1/2}
\]

As we can see from above diagram (Figure 1), frequencies are in range of ELF waves. Also it is seen that with increase of altitudes, frequency will increase.

With respect to diagram, \( \text{NO}^- \) ionic frequencies are more than the frequencies of \( \text{O}_2^+ \) ion species due to it
more mass.

**Alfven Waves**

With dispersion relation (Kelly, 1989; Chen, 1974).

\[ \omega = k \frac{B_0}{(\mu_0 \rho)^{\frac{1}{2}}} \]  \hspace{1cm} (10)

Alfven wave is the second type of ionic waves. We put k and density of different altitudes in equation to obtain frequency \( \omega \).

As can be seen from diagram (Figure 2), range of Alfven waves is placed in range of ELF waves. As we can observe with increase of altitude, frequency will increase. Increasing trend is like previous diagram. In addition to mass in denominator, density in denominator of dispersion relation is influential.

**Magneto-Sonic Waves**

The third type of ionic waves is magneto-Sonic waves. For calculating frequency, we replace Alfven velocity and ionic velocity in different altitudes (Moore et al., 2006; Chen, 1974).

\[ \omega = kc \left( \frac{V_s^2}{c^2} + \frac{V_A^2}{V_s^2} \right)^{\frac{3}{2}} \]  \hspace{1cm} (11)
As can be seen from diagram (Figure 3), magneto-sonic frequencies also are placed in range of ELF waves. The trends of variations in two previous diagrams are so that with increasing altitude, frequency of ions will increase.

CONCLUSION REMARKS

The results indicated that ionic, Alfven and Magneto-Sonic waves are placed in range of ELF waves (10-1000 Hz). In result, we can study dispersion relation of ionic waves in E layer plasma in the range of ELF frequency. In drawn diagrams, NO+ frequency is more than O2+ frequencies due to its lower mass value.

REFERENCES

Bam Juhan, Trueman RA (1380). principles of spatial plasma physics, translation by Dr. Mahmood Moslehi Fard, Dr Behroz Salehi Pour, press of Tabriz university

Chen FF (1974). Introduction to plasma physics, plenum

Gelant, Zilenski, Sakharov, plasma physics, translation, Dr Mahmood

Ghoran Nevis(1373). press center of Islamic Azad University


Diego,


412.

Lehtinen1 NG, Umran S. Inan1 (2008). Received 30 October; revised

16 December 2008; accepted 2 January 2009; published 12

February 2009.

Milikh GM, K Papadopoulos, H Shroff, CL Chang, T Wallace, EV

Mishin, M Parrot, JJ Berthelier (2008). Received 8 May; revised 28

July 2008; accepted 6 August 2008; published 13 September 2008.

Moore RC, US Inan, TF Bell, EJ Kennedy (2006). Received 8

September; revised 21 December 2006; accepted 19 January 2007;


Parrot M, J Manninen, O Santoli’k, F Ne’meč, T Turunen, T Raita, E

Macú’s’ova (2007). Received 9 May; revised 19 August 2007;

accepted 4 September 2007; published 6 October 2007.

Primdahl F (1986). Polar ionospheric E-region plasma stabilization and

Electron heating by wave – induced enhancement of the electron